

S.-T. Yau College Student Mathematics Contests 2023
Oral Exams in Geometry and Topology

Team (Solve 2 out of 3 problems)

1. Prove that there does not exist a compact 5-dimensional manifold W with boundary such that its boundary $\partial W = \mathbb{C}P^2$.

2. Consider the Lorentzian metric

$$(r-1)du \otimes du + \frac{1}{1-r}dr \otimes dr + r^2d\theta \otimes d\theta$$

defined on $\{(u, r, \theta) \mid -\infty < u < \infty, 0 < r < 1, 0 \leq \theta < 2\pi\}$. Can the metric be smoothly extended to $r = 0$ and $r = 1$.

3. Let M^n be an $n(\geq 2)$ -dimensional closed submanifold in the Euclidean space \mathbb{R}^{n+p} . Show that

$$\int_M H^n d\mu \geq \text{Vol}(\mathbb{S}^n).$$

Here H is the mean curvature norm of M , and \mathbb{S}^n is the unit n -sphere.